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Research Article

Evaluating Mathematical Reasoning Competence: In-Service Teacher Training in Mathematical Literacy Given via Distance Education

Zeynep Özaydın[1] **Çiğdem Arslan**[2]

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Abstract:

The purpose of this research is to examine the mathematical reasoning competencies of mathematics teachers. As part of the research, mathematics teachers received in-service training in mathematical literacy. Following the training, classroom practices were conducted by these teachers using course modules designed in alignment with the training content. Due to the COVID-19 pandemic, the in-service training and training practices were conducted via online meeting platforms through distance education. In this research, conducted using the case study method, data were collected through open-ended reasoning tests and a basic concept of mathematical reasoning test. The research found that in-service teacher training and training practices delivered through distance education in the field of mathematical literacy had a positive impact on the mathematical reasoning competence of mathematics teachers. While most of the teachers showed a decrease in their scores on the reasoning test administered after in-service training, they achieved their highest score after training practices they implemented in their own classes. After the training practices, it was observed that teachers' perceptions of mathematical reasoning were at a high level.

Keywords: Distance education, in-service training, mathematical literacy, mathematical reasoning, mathematics teachers.

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[1] Ph.D. Student, Bursa Uludağ University, Bursa, Türkiye, zeynepozaydin@uludag.edu.tr
https://orcid.org/0000-0003-1768-3963
[2] Prof. Dr., Bursa Uludağ University, Bursa, Türkiye, arslanc@uludag.edu.tr
https://orcid.org/0000-0001-7354-8155

INTRODUCTION

Mathematics education provides individuals with a broad range of knowledge and skills that will help them understand the physical world and social roles. It also provides a language analyze, explain, predict, and solve problems from a variety of experiences (Ministry of National Education [MoNE], 2005). The understanding of "mathematical literacy" is adopted as the main purpose of mathematics education (Özgen & Kutluca, 2013; Widjaja, 2011), and this concept is discussed in the context of innovations in mathematics education (Özgen & Kutluca, 2013). Mathematical literacy is the ability to formulate, use, and interpret mathematics in a variety of contexts and to recognize the role mathematics plays in the world (Organisation for Economic Co-operation and Development [OECD], 2013; 2017; 2019). According to Wijayanti & Waluya (2018), mathematical literacy is an ability that enables students to use their mathematical knowledge to solve problems they encounter in daily life.

An essential aspect of mathematical literacy is the notion of "mathematical competence" (Maracci, 2021). Mathematical competencies encompass a comprehensive mastery of mathematics, which includes the ability to solve both mathematical and non-mathematical problems (Niss & Jablonka, 2014). The mathematics frameworks of the Programme for International Student Assessment (PISA), an international student assessment program that extensively covers mathematical literacy, deal with mathematical competencies under seven headings (OECD, 2003; 2013; 2017; 2019). These are "mathematical modeling, problem posing and solving, mathematical reasoning, representation, communication, using symbolic, formal and technical language and operations, using mathematical tools.". Mathematical reasoning is one of the three competencies that are at the top of mathematical competencies (Niss & Højgaard, 2019).

Mathematical reasoning is applied across various stages and activities involved in mathematical literacy (OECD, 2013). Mathematical reasoning is clearly included in the most accepted definition of mathematical literacy in the literature and made by the OECD as "Mathematical literacy includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena" (OECD, 2013, p.25). Similarly, Colwell & Enderson (2016) argue that mathematical literacy is a higher order thinking skill, such as analytical thinking and reasoning skills, while De Lange (2003) states that mathematical literacy requires a concentration on reasoning, thinking, and interpretation. The definitions of mathematical literacy in the literature highlight that mathematical reasoning is a key competence critical for it.

In addition to being one of the primary mathematical competencies (Altun, 2020b), mathematical reasoning, a key skill in the field of mathematics (MoNE, 2024), is an essential requirement across all areas of education (Umay & Kaf, 2005). Mathematical reasoning is a cognitive process that involves people reaching conclusions by producing their own knowledge (Kurtz et al., 1999) and is closely related to understanding mathematics (Ev-

Çimen, 2008). It is a natural part of learning levels, from a more general perspective, it is a natural part of all human activities (Venkat et al., 2009).

Given the importance of mathematical reasoning for both mathematics education and mathematical literacy, it is necessary to prepare environments for the development of this skill (MoNE, 2009; Öz & Işık, 2017). Ensuring the development of mathematical reasoning, which is inherent in education, is a common goal in every field (Altıparmak & Öziş, 2005). It is considered essential to raise awareness among students about the value of mathematical reasoning skills, which ease life (MoNE, 2009). Teachers play a crucial role in fostering the development of mathematical reasoning in students by creating supportive environments and raising awareness about its importance.

Considering the great influence of teachers on students' performance (Jahangir et al., 2012), mathematics teachers need to support reasoning and proof skills to achieve high levels of mathematical knowledge and comprehension in students (Yackel & Hanna, 2003). For this reason, teachers should make mathematical reasoning the focus of their classrooms to enhance students' reasoning skills (Ayele, 2017). However, research on teachers' knowledge and understanding of mathematical reasoning indicates that teachers require support to foster and assess various aspects of this competence (Blanton & Kaput, 2005; Bozkuş & Ayvaz, 2018; Jazby & Widjaja, 2019; Loong et al., 2013; Loong et al., 2018). It is possible to provide this support to teachers through professional development programs, distance education, in-service, and pre-service training. Considering the role of teachers in student achievement, researchers are constantly investigating which components constitute quality teaching to make teacher education and professional development programs, along with education policies, more effective and efficient (OECD, 2016). Teachers develop a positive attitude towards such training, but there is a need for studies to test the effectiveness of such training (Karasolak et al., 2012).

This research focuses on the impact of distance in-service training and related practices in mathematical literacy on mathematics teachers, specifically enhancing their skills in mathematical reasoning. The significant role of mathematical reasoning in mathematics education and its importance in mathematical literacy make it a proficiency demanded by various mathematics education curricula worldwide (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2017; MoNE, 2013; New Jersey Mathematics Curriculum Framework [NJMCF], 1996). This research prioritizes examining the level of reasoning competence in teachers who are curriculum practitioners.

Significance of the Study

In Turkey, mathematics curricula vary in their emphasis on the pivotal role of teachers. For example, the "Ministry of National Education Primary School Mathematics Curriculum and Guide for Grades 6-8" published in 2009, explains the role of teachers in the classroom in detail (MoNE, 2009). Other programs published since then (MoNE, 2013; MoNE, 2018) do not address the specifics of teachers' roles in the learning environment.

Indeed, in the Turkey Century Maarif Model, shaped by a holistic approach to education in line with contemporary trends in the 21st century, the duties of teachers have been reconsidered in detail (MoNE, 2024). The shift in emphasis on teacher roles in mathematics curricula over time demonstrates the need for continuous support for teachers to adapt to changing expectations. Considering the multifaceted roles assigned to teachers in the 21st-century educational approach—not only as knowledge transmitters, but also as student guides, technologically equipped, problem posers, problem solvers, critical and creative thinkers, values education supporters, classroom leaders, researchers, and lifelong learners—, it is crucial that teacher training be regularly updated and provided to reflect current developments. This should occur not only before but also throughout their service.

At the same time, in today's changing and developing world, raising mathematically literate students has become a necessity. Given this need, there is a high expectation for teachers to have an adequate understanding of how to integrate the concept of mathematical literacy into their teaching practice whenever and wherever it is required (Doyle, 2007). These expectations, along with the centralization of mathematical literacy in teaching, have necessitated in-service training for mathematics teachers. In many countries, decreasing school budgets have made it difficult for teachers to participate in professional development activities (OECD, 2016). Within the scope of the relevant research, in-service teacher training in mathematical literacy, which was carried out on a voluntary basis by both educators and participants, responds to this challenge.

On the other hand, Bansilal et al. (2015) expressed the importance of planning and implementing educational content so that education systems, through in-service teacher training, can benefit students. In the relevant research, course modules prepared in accordance with the content of in-service teacher training were used in the classes of teachers who participated in the training. Therefore, the information obtained from teacher training being reflected in the education and training process makes the research important.

This research examines teacher training and training practices in the field of mathematical literacy in the context of "mathematical reasoning". For this reason, it is necessary to mention the importance of "mathematical reasoning" for mathematical literacy. "Mathematical reasoning", which is one of the mathematical literacy competencies, clearly takes its place in the definition of mathematical literacy in PISA sources (OECD, 2013, p.25). In the PISA 2022 Mathematics Framework, mathematical reasoning has been focused on as a fundamental aspect of mathematical literacy (OECD, 2023). Moreover, being mathematically literate depends on the ability to apply mathematical reasoning (Santoso & Sari, 2025). From an educational perspective, it has been determined that students who engage in tasks requiring mathematical reasoning have a higher tendency to develop mathematical literacy skills (Lestari et al., 2021). When it comes to engaging students in tasks that require reasoning, the concept of the "teacher", who is undoubtedly the implementing element of education, comes to the fore.

The National Council of Teachers of Mathematics states that when teachers focus on reasoning, students' learning of mathematics will reach a high level (NCTM, 2000); it also mentions the active role of the teacher factor in making reasoning competence functional. Despite reasoning being a competence included in mathematics curricula around the world, research has revealed that teachers struggle to understand, teach, and evaluate mathematical reasoning (Loong et al., 2013; Loong et al., 2018; Clarke et al., 2012; Davidson vd., 2019). When teachers understand how to create opportunities for their students to reason and how to evaluate these reasoning processes, they will be more likely to incorporate reasoning into their mathematics lessons (Sullivan & Davidson, 2014). It is necessary to gain a better understanding of how teachers perceive the role of reasoning in mathematics teaching and assessment practices (OECD, 2023). In the relevant research, teacher training and training practices in the field of mathematical literacy were deemed important because they made positive contributions to teachers' efforts to understand, teach, and evaluate mathematical reasoning.

Another factor that makes the research important is the low number of studies with teachers compared to the studies conducted with students in the literature on mathematical literacy, both articles (Ülger et al., 2020; Erdoğan & Arslan, 2023) and graduate theses (Arslan et al., 2021; Coşkun-Şimşek et al., 2023). Therefore, it is thought that the research will contribute to the literature on mathematical literacy through the use of its sample type.

Purpose and Problems of the Study

The aim of the research is to reveal the effect of in-service teacher training provided through distance education in the field of mathematical literacy and the use of course modules prepared in accordance with the content of the education on the mathematical reasoning competence of mathematics teachers. The problem statement of the research was determined as *"What impact does distance in-service training in mathematical literacy have on the mathematical reasoning competence of mathematics teachers?"*. The sub-problems of the research are as follows: (i) How do teachers perform on reasoning tests? (ii) What are the perceptions of mathematics teachers who have participated in training and training practices about mathematical reasoning?

METHOD

Research Model

This research was conducted using a case study design, one of the qualitative research methods. Qualitative research is a process in which events are observed in real life and presented realistically and holistically (Yin, 2014; Yıldırım & Şimşek, 2018). In qualitative research, data are generally collected from the participants' own environments and interpreted by the researcher through an analysis that derives generalizations from specific cases (Creswell, 2017). Case studies, one of the qualitative research methods, allow us to investigate the effectiveness of an educational process and examine the reasons this process may or may not be effective (Leymun et al., 2017). The information contained in educational research is process-dependent; it includes information about how the research group is affected in the process. Furthermore, case studies are used when the research problems are related to the process (Rose et al., 2015). This research is suitable for the case study design, which is one of the qualitative research methods, because it presents the data collected in the participants' living spaces in a holistic manner by analyzing them to generalize from specific situations; and reveals the effectiveness of teacher training and practices in the field of mathematical literacy in the context of reasoning through process-oriented research problems.

Participants

The study group of this research consists of six middle school (5th, 6th, 7th and 8th grade level) mathematics teachers who have regularly participated in teacher training and training practices in the field of mathematical literacy. The selection of the study group was carried out according to the criterion sampling method. In the criterion sampling method, all cases that meet the criteria determined by the researcher are included in the study group (Marshall & Rossman, 2014). In the current this research, the criterion for selecting teachers for the study group was determined as "having participated in both mathematical literacy training and training practices and continuing their participation consistently". Teachers work in five state schools in four districts of Bursa in Turkey. Detailed information about the teachers who constitute the study group is provided in Figure 1.

Code: T1 Gender: Female	Code: T2 Gender: Female	Code: T3 Gender: Female
She has 19 years of professional experience. The middle school offers classes at all grade levels. She has not received any training in mathematical literacy before. She stated that to improve her mathematical literacy, She included the solution of vital problems in her class, the questions she used in her lesson were generally test-type, and she always started her lessons with activities.	She has 8 years of professional experience. The middle school offers classes at all grade levels. She took part in the 6^{th} grade mathematical literacy question writing workshop at the Bursa Measurement and Evaluation Center. She stated that she included real-life problems in her classroom to improve mathematical literacy, and that she paid attention to the use of real data by her students in problem-solving studies.	She has 15 years of professional experience. The middle school offers classes at all grade levels. She has not received any training in mathematical literacy before. To improve mathematical literacy, she stated that she had brainstorming activities in her classroom before starting the subjects.
Code: T4 Gender: Male	**Code: T5 Gender: Female**	**Code: T6 Gender: Female**
He has 17 years of professional experience. The middle school offers classes at all grade levels. He has not received any training in mathematical literacy before. He stated that he made problem-solving practices that measured skills in his classroom to improve mathematical literacy.	She has 6 years of professional experience. The middle school teaches at the 5^{th}, 7^{th} and 8^{th} grade levels. She has not received any training in mathematical literacy before. She stated that to improve mathematical literacy, she carried out studies for students to read and understand Turkish texts in her classroom and carried out mathematical problem-solving studies.	She has 10 years of professional experience. The secondary school teaches at the 5th, 7th and 8th grade levels. She has not received any training in mathematical literacy before. She stated that she used materials, contextual explanations and dynamic software in her lesson to improve mathematical literacy.

Figure 1. Detailed Information about the Teachers in the Working Group

Data Collection Tools

The tools used to collect data in the research are "Reasoning Tests (RT1, RT2, RT3)" and "Basic Concept of Mathematical Reasoning Test". The questions in the "Reasoning Tests (RT1, RT2, RT3)" were selected and compiled by the researchers from various sources

(Altun, 2020a; Altun, 2021a; Altun, 2021b; Altun, 2021c; Consortium for Mathematics and Its Applications [COMAP], 2008 pp.129,138) and were applied before teacher training, after teacher training and after the training practices. Each test consists of four open-ended questions. A sample question from each test is provided in Appendix 1. The questions were presented for the opinion of six experts in the project team. One of the experts who works in the field of mathematical literacy and competencies is a professor, two are associate professors, and three are doctors. Opinions were gathered from experts about whether the questions allow the observation of the reasoning competence of the individual, and whether the tests are equivalent to each other. At the same time, they were asked to make suggestions for making the questions more insightful. Considering the suggestions of the experts, the questions have been revised and deepened.

The "Basic Concept of Mathematical Reasoning Test", which is applied at the end of the training practices, was prepared by the researchers to reveal the mathematical reasoning perceptions of the teachers after the training and training practices. The prepared questions were examined by the field expert in terms of whether they were relevant to the field of reasoning or not. It was decided that all the questions of the open-ended test were aimed at the concept of mathematical reasoning. The questions in the test are "(i) What is mathematical reasoning? (ii) How do you know that your students are reasoning? (iii) What are the indicators of reasoning?"

Data Collection Process

This research was carried out during the 14-session in-service teacher training and practices of the project titled "Increasing the Level of Mathematical Literacy with Dual Focus Teaching Model (Cift Odaklı Öğretim Modeli – COM). In-service teacher training, consisting of five main parts (i. what is mathematical literacy, ii. choosing and writing a mathematical literacy question, iii. adapting the teaching content to mathematical literacy, iv. introducing the COM and v. module development, planning, and implementation suitable for COM), was organized and carried out in a structure that would deepen the understanding of mathematical literacy (Altun et al., 2024). In-service teacher training and practices were carried out in the form of distance education on online meeting platforms due to the COVID-19 pandemic.

Data were collected in stages. After determining the teachers who would participate in the training, "RT1" was administered to teachers before the training and "RT2" afterward. Teachers who participated in the teacher training and wanted to join the training practices were selected on a voluntary basis. The practice teachers used the COM course modules (prepared by the project team) that matched the training content in an online classroom environment for an educational training period. At the end of this period, "RT3" and "Basic Concept of Mathematical Reasoning Test" were administered to practice teachers. The data collection process is shown in Figure 2.

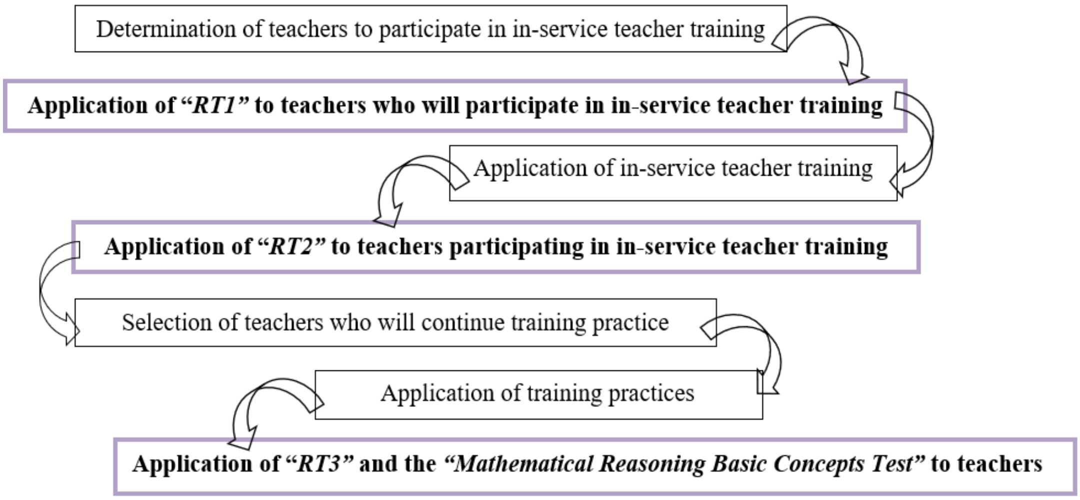

Figure 2. Data Collection Process

The Role of the Researcher

The researchers are in the team of the project in which the research is produced. They took an active role in every stage of in-service teacher training in the field of mathematical literacy carried out within the scope of the project. In line with the content prepared by the project coordinator, the materials and documents were made ready for implementation over 14 sessions of the teacher training. The organization of the online sessions, the sharing of participation information with teachers, and conducting the training sessions were carried out by the researchers.

In qualitative research, a researcher is a person who spends time in the field of data collection, communicates directly with the study group, witnesses the experiences of the study group, and reflects the perspective gained in the field of data collection to the analysis of the data (Yıldırım, 1999). In this research, the participating teachers were able to communicate with the researchers whenever they needed and were able to find solutions and answers to their problems. As a result of this situation, the researchers became acquainted with the participating teachers closely.

After the teacher training, the researchers took an active role in preparing the COM course modules aligning with the training content and delivering them to the participating teachers. The researchers applied the "Reasoning Tests (RT1, RT2, RT3)" and the "Basic Concept of Mathematical Reasoning Test" and collected the data. The data collected were analyzed by the researchers.

Data Analysis

For the first research problem, RT1, RT2, and RT3 were applied. The obtained data were analyzed and scored using the "Reasoning Competence Rubric (RCR)" developed by Özaydın & Arslan (2022). To apply RCR in data analysis, the criteria within RCR that could be observed through the test questions were first identified. The criteria established by the expert were kept numerically consistent across all tests, ensuring that the maximum obtainable score was equal for each test (Figure 3). Appendix 2 includes justifications for the criteria in a sample problem, and the rubric indicating how many points will be received from each criterion in different situations. In determining the criteria, the percentage of agreement between the researchers was calculated using the reliability formula proposed by Miles & Huberman (1994). By determining the criteria, the percentage of agreement between the researchers was calculated to be 90.2%, and this rate is sufficient for the evaluation results to be considered reliable (Miles & Huberman, 1994; Şencan, 2005).

	RT1				RT2				RT3			
	Question 1	Question 2	Question 3	Question 4	Question 1	Question 2	Question 3	Question 4	Question 1	Question 2	Question 3	Question 4
K1							+	+	+			
K2	+	+		+	+	+		+	+	+	+	+
K3			+					+				
K4	+	+	+	+		+	+			+		+
K5			+		+	+	+		+			
K6	+	+	+	+	+	+	+	+	+	+	+	+
K7							+					+
K8	+	+			+		+		+	+	+	+
K9	+	+	+	+	+	+		+	+		+	+
K10	+	+			+	+			+	+	+	
K11		+	+	+		+					+	
K12												
Max Score	12	14	12	10	12	14	12	10	14	10	12	12
Min Score	0	0	0	0	0	0	0	0	0	0	0	0
Sum Max Score	48				48				48			
Sum Min Score	0				0				0			

Figure 3. Scores that can be Obtained for the Criteria in the Tests

For the second research problem, "Basic Concept of Mathematical Reasoning Test" was applied. The data obtained were analyzed using the mathematical reasoning perceptions framework (Figure 4) defined by Herbert et al. (2015) to reveal the perceptions of mathematical reasoning of mathematics teachers who participated in training and training practices. The framework consists of hierarchical categories. Loong (2014) states that this framework can be a tool for assessing teachers' awareness of different aspects of reasoning over time and comparing their perceptions. This framework facilitates the monitoring of perceptions to assess the effectiveness of learning opportunities (Herbert et al., 2015).

Category	Perception of mathematical reasoning
Category A	Reasoning is perceived to be thinking
Category B	Reasoning is perceived to be communicating thinking
Category C	Reasoning is perceived to be problem solving
Category D	Reasoning is perceived to be validating thinking
Category E	Reasoning is perceived to be forming conjectures
Category F	Reasoning is perceived to be using logical arguments for validating conjectures
Category G	Reasoning is perceived to be connecting aspects of mathematics

Figure 4. Teachers' Perceptions of Mathematical Reasoning (Herbert et al., 2015)

The Role of the Researcher Validity and Reliability of Data

Validity is related to the accuracy of research results (Baltacı, 2019). The researcher's presentation of the data to expert review contributes positively to the validity (Denzin & Lincoln, 2008; Merriam, 1998). Expert opinions were consulted at every stage of the relevant research, such as the preparation of data collection tools, the preparation and selection of tools in data analysis, and the interpretation and confirmation of the findings. Miles and Huberman (1994) emphasized that to ensure validity in qualitative research, the researcher must be consistent in the analysis and interpretation of data. Clearly expressing the scoring analyses (Appendix 2-3) for the rubric used in the study and sharing these with the reader provide evidence of the researcher's consistency in data analysis and interpretation. Merriam (1998) stated that long-term observations in case studies enhance validity. In this case, it can be said that the validity is ensured by collecting the data of the relevant research in three stages (pre-training, post-training, post-training practices) over a period of approximately nine months.

Reliability is related to the reproducibility of research results. Because human behavior is highly variable, measuring reliability poses challenges in social science research (Miles & Huberman, 1994). LeCompte & Goetz (1982) stated that clearly expressing the position of the researcher in the research increases reliability. Yin (2014) indicated that supporting the research with documents enhances its reliability. Additionally, Miles & Huberman (1994) noted that explaining in detail the data collection tools, the data collection process, and how the data was analyzed further increases reliability In this research, reliability was ensured by clearly stating the role of the researcher, supporting the findings with photographs of teacher responses, and providing a detailed explanation of the data collection tools, the data collection process, and how the data were analyzed.

Ethical considerations

Ethical Review Board: Bursa Uludağ University Research and Publication Ethics Committees, Social and Human Sciences Research and Publication Ethics Committee

Date of Ethics Review Decision: 25.10.2018

Ethics Assessment Document Issue Number: 2018-09

RESULTS

The first sub-problem of the research is "How do teachers perform on reasoning tests?". It is possible to observe the findings related to this problem through the scores of the teachers with the codes T1, T2, T3, T4, T5, and T6 on the "Reasoning Tests (RT1, RT2, RT3)". After the teachers' answers to the reasoning tests were interpreted according to the criteria in the RCR, they were scored accordingly. Details of the process analyzing two sample teacher responses and their scores, against the criteria, are included in Appendix 3. The scores were reflected in the graph, thus revealing the trend in the teachers' scores from the reasoning tests. Teachers' scores on reasoning tests are presented in detail in tables, along with the test questions. Subsequently, graphs of the scores were organized to illustrate the progression of the results. The scores of the teachers from the questions in the tests are given in Table 1.

Table 1

Teachers' scores on RT1, RT2, and RT3

	RT1					RT2					RT3				
Tecaher Code	Question 1	Question 2	Question 3	Question 4	**Sum**	Question 1	Question 2	Question 3	Question 4	**Sum**	Question 1	Question 2	Question 3	Question 4	**Sum**
T1	6	9	7	7	**29**	9	1	10	0	**20**	10	8	5	7	**30**
T2	10	8	6	8	**32**	9	12	8	9	**38**	13	10	10	12	**45**
T3	10	13	4	9	**36**	8	12	8	7	**35**	11	10	11	12	**44**
T4	4	14	12	6	**36**	12	7	6	8	**33**	14	8	11	8	**41**
T5	10	9	9	9	**37**	12	8	6	7	**33**	13	10	9	11	**43**
T6	8	7	10	10	**35**	12	8	8	6	**34**	12	10	9	12	**43**

Figure 5, prepared to show the progression of the test scores of all teachers, is given below.

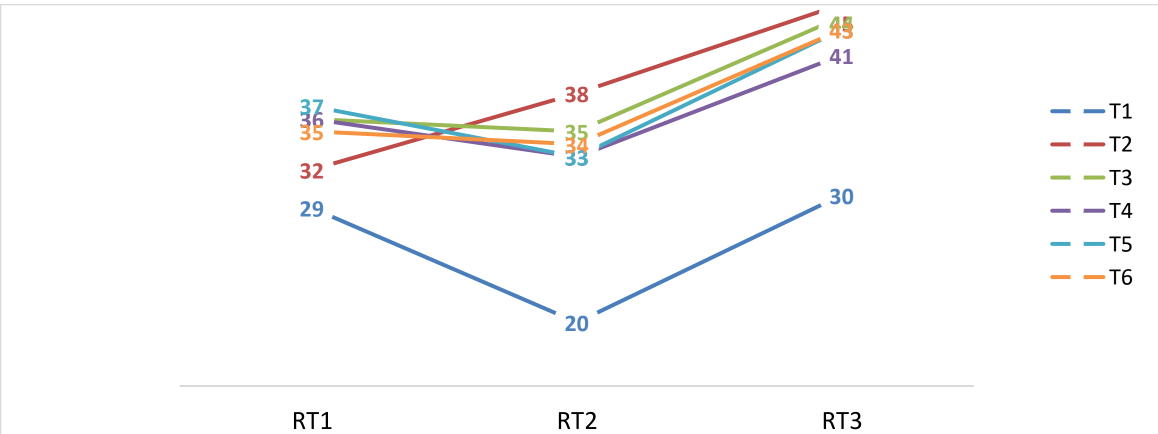

Figure 5. The Course of Teachers' Scores from RT1, RT2 and RT3

Figure 5 shows that most teachers experienced a score decrease from RT1 to RT2, and all teachers achieved the highest score on RT3. T2 is the only teacher who has continuously increased her score without experiencing a decline. At the same time, T1 scored lower than the other teachers on all the tests, falling below the area of pile-up on the graph.

The second sub-problem of the research is "What are the perceptions of mathematics teachers who have participated in training and training practices about mathematical reasoning?". The findings related to this problem were obtained from the "Basic Concept of Mathematical Reasoning Test". The data were analyzed using the mathematical reasoning perceptions framework put forward by Herbert et al. (2015). Table 2 shows the perceptions of reasoning that emerged from the teachers' answers to the questions "What is mathematical reasoning? How do you know that your students are reasoning? What are the indicators of reasoning?"

Table 2

Teachers' Perceptions of Mathematical Reasoning

Category	"Perception of mathematical reasoning"	f	%
A	"Reasoning is perceived to be thinking"	6	24
B	"Reasoning is perceived to be communicating thinking"	1	4
C	"Reasoning is perceived to be problem solving"	5	20
D	"Reasoning is perceived to be validating thinking"	2	8
E	"Reasoning is perceived to be forming conjectures"	1	4
F	"Reasoning is perceived to be using logical arguments for validating conjectures"	6	24
G	"Reasoning is perceived to be connecting aspects of mathematics"	4	16
	Sum	25	100

As shown in Table 2, perceptions in categories A and F were observed most frequently, appearing in 24% of responses. This indicates that all six teachers in the study group demonstrated perceptions associated with categories A and F. Category C was the second most observed, present in 20% of responses, while category G was observed in 16% of responses, ranking third. Category D appeared in 8% of responses, placing it fourth. Finally, categories B and E were the least observed, identified in the views of two teachers, with perceptions in these categories comprising 4% of responses.

Some of the teachers' opinions are given below. In addition, it was explained which perception category included the teachers' opinions. The teacher's opinion in Figure 6 is included in category A because it mentions the act of thinking, and in category F because it mentions the act of argumentation.

How do you know that your students are reasoning? What are the indicators of reasoning?
Analytical thinking, argumentation.

Figure 6. Teacher's Opinion Included in Categories "A" and "F"

The teacher's opinion in Figure 7 was included in category F for mentioning mathematical relationships, D for mentioning verifying an idea by justifying it, E for mentioning generalizations, A for mentioning thinking, and B for mentioning the necessity of hearing one's thoughts out loud.

How do you know that your students are reasoning? What are the indicators of reasoning?
Analysis, verification and generalization. If the student is able to notice the mathematical relationships by making the student hear aloud in the answer he gives, it means that the student is using the analysis step. If he approves an idea by justifying it, he confirms it. In general, students have very little access to the level of generalization, and at this point, we, the educators, have important duties. In order to understand that students are reasoning, we need to hear out loud what they are thinking in their heads.

Figure 7. Teacher's Opinion Included in Categories "A", "B", "D", "E" and "F"

The teacher's view in Figure 8 is included in category C because it mentions problem-solving. On the other hand, the expression "blending new information in the mind" is viewed as a situation related to the restructuring of knowledge (category G).

What is mathematical reasoning?
To blend the new knowledge acquired with the accumulation of knowledge in the mind. Mathematical reasoning, bringing this to mathematics class, using mathematical skills to solve problems in daily life and comparing causes and effects.

Figure 8. Teacher's Opinion Included in Categories "C" and "G"

The teacher's opinion in Figure 9 was included in category C because it mentioned different solution methods and in category F because it mentioned connections.

How do you know that your students are reasoning? What are the indicators of reasoning?
Can they use different solutions? Are their situations able to connection events correctly? Is it able to compare correctly? I'll look at those.

Figure 9. Teacher's Opinion Included in Categories "C" and "F"

The teacher's opinion in Figure 10 was included in category A because it mentioned thinking, and in category C because it mentioned problem solving.

What is mathematical reasoning?
Reasoning, in my opinion, means that the student thinks about the solutions and decides how to solve the problem.

Figure 10. Teacher's Opinion Included in Categories "A" and "C"

The teacher's view in Figure 11 was included in category C because it mentioned problem solving and in category D because it mentioned mathematical bases. On the other hand, the expression "mathematical processes" was included in category F because it was thought to evoke the use of step-by-step logical arguments.

What is mathematical reasoning?

We can think of it as reasoning and using some mathematical processes while solving the problems we encounter in life with mathematical knowledge. And at the same time, we may have made mathematical reasoning by providing mathematical bases for the solution of the problem.

Figure 11. Teacher's Opinion Included in Categories "C", "D" and "F"

Considering the hierarchical structure of the categories, the fact that teachers' perceptions have the highest frequency in category F as well as category A is a positive finding in terms of teachers' reasoning perceptions. That is, all teachers expressed an opinion that can be included in category F.

DISCUSSION AND CONCLUSION

The purpose of this research is to reveal the effects of in-service teacher training and training practices provided through distance education in the field of mathematical literacy on the mathematical reasoning competence of mathematics teachers. At the same time, an examination was conducted on the mathematical reasoning perceptions of teachers who participated in training and training practices. Before discussing the results obtained from this research, it is essential to note that reasoning is an individual process and may vary depending on the perspective of the evaluator (Umay, 2003). In addition, the research limitations should be mentioned. Interpreting the discussion, results, and suggestions in this way would be a more accurate approach. The findings of this research are limited to the data obtained from six middle school (5th, 6th, 7th and 8th grade level) mathematics teachers who participated in the project titled "Increasing the Level of Mathematical Literacy with Dual Focus Teaching Model", attended a 14-session teacher training program, and implemented the course modules—prepared in line with the content of the training—in their classrooms over the course of one academic semester. In addition, the long-term retention of teachers' learning was not evaluated, which can also be considered a limitation of the research.

Considering the progression of the scores obtained from the reasoning tests, most teachers exhibited a decrease in their scores on the reasoning test administered after the in-service teacher training. The reason for this situation may be that teacher education is provided through distance education. Karaevli & Levent (2022) concluded that teachers have limited opportunities for interactive and lasting learning in remote education, due to the inability of online training to facilitate immediate practical applications compared to face-to-face instruction. At this point, continuous evaluation of distance education applications and designing new training in line with these evaluations will benefit the quality of the training to be given (Taşlıbeyaz et al., 2014). However, it should be considered that the beliefs of the participants regarding distance education are also an important factor in achieving its desired quality (Horzum & Canan Güngören, 2012).

At the same time, designing professional learning processes that will improve teachers' understanding of reasoning is reportedly extremely complex (Rasmussen & Marrongelle, 2006). Teaching reasoning is difficult because it requires identifying complex thought processes (Rogers & Steele, 2016). In this case, it will take time for teachers' reasoning scores to improve, as this was also observed in the present research. Teachers reached the highest score in the reasoning test applied at the end of the training sessions. Frith & Prince (2006) state that it would be useful to design the content of education as a social practice when planning in-service teacher training in the field of mathematical literacy. The literature suggests that in-service training should encompass not only theoretical instruction but also practical applications to effectively transfer this training into the classroom environment (Bansilal et al., 2015; Bozkurt, 2019; Loong et al., 2017; Ülger, 2021). Loong et al. (2017) observed improvement in both teachers' and students' perceptions and understanding of mathematical reasoning because the teachers participated in a professional learning program and efficiently implemented the learning program in their classrooms.

It was observed that the teacher who scored lower than the other teachers in all the reasoning tests and was below the region with the accumulation in the graph, was the teacher with the most professional experience. Botha et al. (2013) found that teachers with more teaching experience performed mathematics literacy practices more efficiently. However, it should be considered that the in-service training and training practices within the scope of this research were carried out through distance education. Horzum et al. (2012) state that as teachers' professional experience increases, their belief in distance in-service training decreases and the difficulties they experience increase.

If the general research question is answered with the results discussed thus far, the impact of in-service teacher training and training practices provided through distance learning in the field of mathematical literacy on mathematics teachers' mathematical reasoning competence is positive. This positive result revealed by the current study is both desired and expected. Herbert et al. (2022) found that teachers who participated in a professional learning program designed to help integrate reasoning into mathematics lessons increased their awareness of various aspects of mathematical reasoning. Similarly, Herbert and Bragg (2021) report that teachers who participated with their peers in a professional learning program designed to identify reasoning and develop strategies for applying reasoning to mathematics lessons reported experiencing positive effects. Based on the results obtained from the current study and those in the literature, it can be argued that teachers' processes of recognizing, interpreting, and integrating reasoning skills into the classroom can be improved through planned professional learning experiences.

On the other hand, Esendemir et al. (2015) reported that preservice elementary mathematics teachers perceived themselves as well-equipped in mathematical reasoning, while Güler and Arslan (2019) found that preservice mathematics teachers lacked awareness of these competencies. Similarly, even though mathematics teachers' conceptualization of mathematical reasoning aligns with the international curriculum (Jeannotte et al., 2020), teachers have a limited understanding of mathematical reasoning (Herbert et al., 2022). Therefore, even if mathematics teachers feel equipped or demonstrate certain success in mathematical reasoning both before and in service, their awareness of the underlying reasoning process is limited. Accordingly, their understanding of mathematical reasoning remains superficial. Given that developing and supporting teachers in mathematical literacy is seen as important and beneficial (Özgen, 2019; Frith & Prince, 2006), the same can be said for reasoning, one of the mathematical literacy competencies. Studies suggesting that teachers need training to develop a deeper understanding of mathematical reasoning (Ayele, 2017; Bozkuş & Ayvaz, 2018; Loong et al., 2013; Öz & Işık, 2020) support this idea.

It has been determined that mathematics teachers' perceptions of mathematical reasoning after following the training practices are at a high level. However, Bozkuş and Ayvaz (2018) indicate that mathematics teachers often lack comprehensive and sufficient knowledge regarding mathematical reasoning, highlighting the need for additional training to enhance their understanding. Therefore, it is expected that the teachers in the study group of the current research, who have received mathematical literacy training and implemented its content through practical classroom applications, exhibit high levels of perceptions of mathematical reasoning. Similarly, Park and Magiera (2020) assert that pre-service mathematics teachers who receive education in mathematical reasoning are better equipped to interpret it broadly, encompassing aspects such as thinking, validating ideas, problem-solving, and synthesizing or making sense of concepts.

When the results obtained within the scope of the study are evaluated from a general perspective in line with the research problems, it is clear that in-service teacher training and training practices in the field of mathematical literacy have positively impacted mathematics teachers' skills and perceptions of mathematical reasoning competence. Mathematical reasoning is an important skill for every individual in every classroom environment in today's century. Therefore, even if teachers are sufficiently knowledgeable about reasoning, they must always receive support to deepen this knowledge to keep pace with the developments of the age and to reach more students in our changing world. For example, it may become necessary to integrate artificial intelligence applications, which have gained popularity in recent years, with reasoning in learning environments. This research has revealed that mathematical literacy is an effective context for teachers to develop and deepen their mathematical reasoning competencies.

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Appendix 1- Sample Questions From Reasoning Tests (RT1, RT2, RT3)

Reasoning Test (RT1) **- Question 1 (Altun, 2021c)**

To determine the order of transactions at the counters, a bank gives a two-digit sequence number to those who queue with an ID number, and a four-digit sequence number to those who queue with a bank card. To give priority to those who queue with cards for transactions at the box office, an ordinary period of "card, card, identity" is followed in calling customers. The numbers in the hands of ten customers waiting in line to be processed in the waiting room are 5324, 78, 5321, 77, 5322, 5323, 79, 80, 81,5325.

a) Since the customer who receives sequence number 77 is called to make a transaction in the first row at the box office, what will be the turn of the transaction for 80? Justify it.

b) Does a customer with a card who enters the bank at that moment change the order in which the customer with the sequence number 80 makes a transaction? Explain.

c) Would you suggest a quadruple period that would accept the customer with the number 78 in the 6th row, provided that the first person called was the card? Support your suggestion mathematically.

Reasoning Test (RT2) **- Question 2 (COMAP, 2008, p.129-138)**

A toy manufacturer produces skateboards and dolls. The production of skateboards requires 5 units of plastic and can be sold for a profit of 1 TL. The production of dolls requires 2 units of plastic and can be sold for a profit of 0.55 TL. The toy manufacturer has 60 units of plastic available.

a) How much skateboard/doll should the toy manufacturer produce to maximize its profits in the presence of so many materials? Justify your solution.

b) It takes 15 minutes to skateboard and 18 minutes to make dolls. The toy manufacturer has 360 minutes available. How much skateboard/doll should the toy manufacturer produce to maximize its profits in the presence of so much time and material (60 units of plastic)? Demonstrate your solution.

c) How do you decide on the accuracy of your solutions? Show me.

Reasoning Test (RT3) **- Question 3 (Altun, 2020)**

Standard paper sizes, commonly used in printing and correspondence, are designated as A0, A1, A2, ..., in descending order of size. Each paper size is a similar rectangle to the others. A4 is also known as letter paper, with dimensions of 21 x 29.7 cm. To obtain a larger one, it is necessary to place two sheets of the same size side by side along the long side.

a) According to this information, what kind of relationship should there be between m and n for a paper with dimensions' m and n to be standard paper size? Show me. How can you make ensure the validity of this relationship?

b) Stained glass (glass painting) will be made on an A3 size glass panel. On the paint tube to be used for the painting process, it is written that it can paint a maximum area of 1092 cm^2. Is this tube of paint enough to paint this glass panel? Your idea should be based on a mathematical solution process.

Appendix 2- Criteria and Rubric for a Sample Problem (RT1-Question 1)

Reasoning Test (RT1) - Question 1

Reasons for selecting the criteria in the problem

a) Determining the order in which customer number 80 will perform a transaction depends on the mathematical solution process (K4).
b) Whether the order of transactions of the cardholder currently at the bank will affect customer number 80 is a contextual problem. The contextual problem should be interpreted in a real-world context (K8), along with explanations supporting the solution (K10).
c) A representation in the four-period format that assumes customer number 78 is considered the sixth in line should be presented (K2). Relationships between contextual and mathematical language should be established during the representation (K9).
During the solution process, the rules provided in the question text should be used correctly (K6).

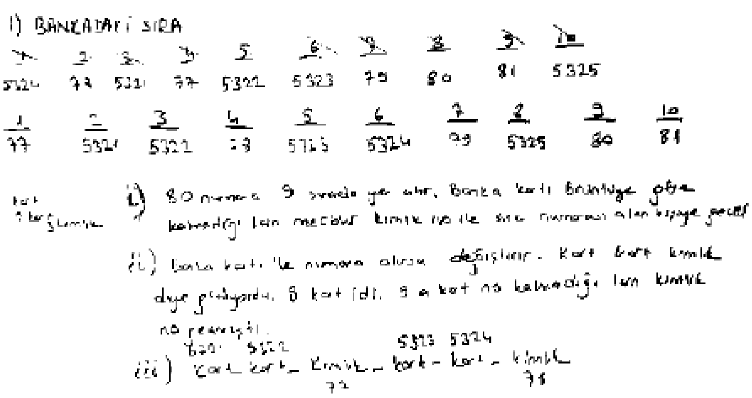
Rubric for the problem

RT1-Q1	0	1	2
K2	There is no representation.	There is a representation that accepts customer number 78 as the 6th row. However, its period is not quadruple.	The quadruple period that assumes accepting customer number 78 as the 6th row is "card-ID-card-card."
K4	There is no resolution process./There is a resolution process, but it is incorrect.	*"77-5321-5322-78-5323-5424-79-5325-(No more cardholders)80"* It was determined that customer number 80 would be the 9th to process the issue, but no justification was provided.	The justification for the resolution process was given as *"because there are no more cardholders."*
K6	The rules provided in the question text were not considered.	The rules provided in the question text were used incorrectly/incompletely.	During the solution process, the rules provided in the question text were used correctly/completely.
K8	There is no contextual interpretation.	Interpretations such as "the order of operations does not change" are incorrect.	*"the order of operations change"* is a correct contextual interpretation based on the mathematical result.
K9	There is no representation.	The relationships between languages in the representations proposed outside the quadruple period are flawed.	The relationships between contextual language and symbolic language in the proposed representation of *"card-ID-card-card"* are correct.
K10	The solution to the problem has not been considered.	The presence of a contextual comment (K8) indicates that a solution has been considered, but there is no explanation.	The following explanation supports the solution to the problem: *"The card-holding customer will queue after 5325, so they will be ahead of person number 80."*

Appendix 3- Sample Analyses (RT1-Question 1)

RT1-Q1	K2	K4	K6	K8	K9	K10	Sum
T2	1	2	2	2	1	2	10
T4	1	0	1	1	1	0	4

T2's answer to Question 1 in RT1



As seen in the photo, the teacher's incorporating the bank order rule into his solutions, without ignoring it throughout the question, demonstrates his understanding of the rules and definitions (K6). She answered correctly in the first item of the question by saying, "Number 80 is in 9th place." She gave a justification for the solution process she carried out as "since there are no customers with bank cards, the queue goes to the person who takes the queue with their ID number" (K4). The answer in the second question, "If the bank takes the number with the card, it changes", indicates that the teacher can interpret in a real-world context (K8). After the teacher decided to change the order, she provided a statement supporting this decision: "The cards were going as card IDs. There were 8 cards. Since there were no card numbers left for 9, the ID number was used" (K10). When we reached the last item of the problem, the teacher proposed a new period - representation - that would accept customer number 78 as the 6th row, but since this period was a terminal rather than a quadruple period, the necessary assumptions were not made (K2). This shows that the relationship between contextual language and the mathematical language in which the problem is conveyed has not been fully established (K9).

T4's answer to Question 1 in RT1

1. Bankada Sıra
   a. Kimlik – Kart – Kart Sıralaması olduğuna göre 80-77= 4. Kimlikle sıra alan $3 \cdot 3 + 1 = 10$ 10. Sırada işlem yapacaktır.
   b. Değiştirmez çünkü sıralamalar kendi içinde yapılmaktadır.
   c. Burada sıranın 78 olması sorunun çözümüne etkisi yoktur. 78 olması kimlikle sıra alındığını gösterir.
   İki durum söz konusudur. (1)Kart – (2)Kart – (3) Kimlik – (4) Kart – (5)Kart – (6) Kimlik
   (1)Kart – (2) Kimlik – (3) Kart – (4)Kart – (5) Kimlik – (6) Kart
   Görüldüğü gibi istene sıra 3. Katı olduğu için sadece ilk durumla 6. Sıra alınabilir.

As seen in Photograph, T4 ignored the fact that the number of customers with debit cards was 5 on the first item of the question and stated that customer number 80 would be the 10th to process. The solution process exists, but it is incorrect (K4). The answer "doesn't change" in the second item of the question is an interpretation in the real-world context, but the interpretation is incorrect and does not depend on the mathematical result (K8). Because there is no mathematical solution, the criterion coded K10 cannot be observed. The examples in which the teacher represented the problem in different ways in the last item of the question are those for which the necessary assumptions were not made – examples with a threefold. The language used in the contextual and mathematical representations does not fully align (K9). Looking at the answers given throughout the question, it appears that the teacher does not fully understand the rules provided in the main text of the question (K6).

Data Availability Declaration

While the primary datasets utilized in this study are not publicly accessible due to certain constraints, they are available to researchers upon a formal request. The authors have emphasized maintaining the integrity of the data and its analytical rigor. To access the datasets or seek further clarifications, kindly reach out to the corresponding author. Our aim is to foster collaborative academic efforts while upholding the highest standards of research integrity.

Author Contributions

Zeynep Özaydın and Çiğdem Arslan spearheaded the conceptualization, designed the research methodology, and supervised the entire article. Zeynep Özaydın was responsible for the data collection, analysis, and interpretation, bringing analytical rigor to the study. Zeynep Özaydın and Çiğdem Arslan took the lead in drafting the manuscript, ensuring its alignment with scholarly standards, and revising it for intellectual depth. All authors collaboratively discussed the results, provided critical insights, and contributed to the final manuscript. They have read, approved, and take joint accountability for the presented work's accuracy and integrity.

Authors' statements on ethics and conflict of interest

Ethics statement: We hereby declare that research/publication ethics and citing principles have been considered in all the stages of the study. We take full responsibility for the content of the paper in case of dispute.

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Biographical notes:

Zeynep Özaydın 1[3]: Zeynep Özaydın is currently pursuing her doctorate. She has published articles and ongoing research on mathematical reasoning, mathematical literacy, mathematical thinking skills, and higher-order thinking skills.

Scopus Author Identifier Number: 58739226400

Web of Science Researcher ID: MCJ-3328-2025

Google Scholar Researcher ID: B4iel5EAAAAJ

Çiğdem Arslan 2: Çiğdem Arslan has published numerous articles in international journals and has also taken part in various projects in different roles, all focusing on problem solving-posing, geometry teaching, mathematical thinking, mathematical anxiety, mathematical reasoning and mathematical literacy.

Scopus Author Identifier Number: 26656549500

Web of Science Researcher ID: C-9729-2019

Google Scholar Researcher ID: 98ckc60AAAAJ

[3] Corresponding Author