Research

Developing a Number Sense-Based Instructional Design to Eliminate Student Errors Based on Mathematical Misconceptions

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Abstract:

The purpose of this research was to develop and evaluate a number sense-based curriculum aiming to eliminate student errors due to mathematical misconceptions. A mixed method research model as one of the qualitative research approaches was adopted in this study aiming to develop a number sense-based instructional design to eliminate student errors arisen from mathematical misconceptions in secondary school students. In preparation stage of the instructional design, 27 seventh grade students studying at a state school in Mersin province were determined as the study group. At the stage of developing and evaluating the instructional design, 24 fifth grade students studying at a state school in Mersin province were determined as the study group. In the process of developing the instructional design, "Needs Analysis Student Interview Form," "Instructional Design Teachers' Board Interview Form," and "Expert Interview Form" were used. When all these results were evaluated together, it would be correct to say that number sense was an important prerequisite for the interpretation and use of mathematical knowledge, and it would be efficient on eliminating errors arisen from the mathematical misconceptions when it was integrated into an effective instructional design. It was possible to suggest that further researches should be conducted for determining the misconceptions of the students related to different mathematics sub-topics.

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INTRODUCTION

Information has recently been changed, developed, and used very rapidly. In fact, the remarkable point here is adaptation into this situation and structuring the information accurately. The key point of this change in knowledge is education. Education that starts in family at birth continues providing individuals with desired behaviors in formal institutions through instructional programs. As of 2005-2006 academic year, the curriculums in our country have been restructured according to the constructivist approach. According to Özdemir and Uyangör (2011), new curriculums have highlighted the cognitive dimension of learning, and emphasis on individualized learning centering the learning needs of the students has been the main vision of the program. All these changes have caused each discipline to adopt different visions and goals. Curriculums include many elements that interact with each other in achieving the goals, but the guide for how to teach is instructional design. According to Fer (2009), if the design of instructional systems focuses on pure teaching instead of the whole education system, it appears as the instructional design. The main purpose of instructional design is to plan, develop, evaluate, and administer the instructional process (Akay, 2017). This is a process which is possible by performing detailed analysis of all of the components that are involved in the teaching process. In cases where the instructional process in the curriculum is not designed effectively, it will be difficult to talk about meaningful learning. According to Şen and Erişen (2002), the basic element of the instructional system is the teachers who are the implementers of the curriculum. The success of a curriculum is directly dependent on the characteristics of the teachers, and it is not disconnected from the instructional designs that teachers benefit as a guide. Instructional design models include the tools to be used in teaching environments to support student learning (Fer, 2009). An instructional design approach should focus on the product within the framework of learner characteristics (Morrison, Ross, and Kemp, 2012). Taking all of the precautions to increase the quality of the product and implementing them in this process will result in the highest level of the efficiency and output. It is very important to benefit from these instructional design approaches while achieving the goals of the curriculum.

As in all curricula, the effectiveness of the teaching process in mathematics curriculum is possible to be improved with the instructional design. There are various interrelated concepts and processes in the discipline of mathematics and teaching of these concepts continues with formal education starting from early childhood. Mathematical operation processes are created through the relationships of concepts with each other. "Although conceptualization in children is a very difficult and slow process, children begin to acquire concepts at the age of one or two. The children at a young age encounter new information every day and either associate this information with pre-existing concepts or develop new concepts" (Arnas, 2005, p.3). According to this, mathematical concepts are the basis for mathematical operations and skills. The concept misconceptions that will occur in the future will be prevented when the students are correctly taught about these concepts at the first time in which they see these concepts. Because the mathematical misconceptions that start

developing in earlier periods will turn into mathematical mistakes (mathematical fallacies) and mislearning when the sophistication of the topics increases. It has been emphasized in revised 2018 primary education mathematics curriculum of the Ministry of National Education (2018) that mathematics education is one of the basic skills and competencies and contributes upon a happy and successful life. Administering mathematics teaching process with an instructional design approach plays an active role in achieving the goals of the curriculum. Considering as a theory, the elements of instructional design were presented in Figure 1.
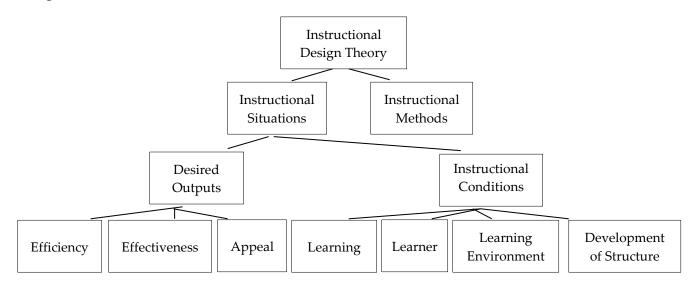

Figure 1. Instructional Design Theory and its Elements

Reference: Reigeluth (1999)

According to Figure 1, an instructional design output should be efficient, effective, and appealing. Moreover, it should guide for the development of education providing necessary instructional conditions. Various teaching models have recently been used. The most frequently used of these instructional design models are specified as ADDIE, Gagne & Briggs, Morrison, Ross & Kemp model (Khodabandelou and Samah, 2012). In this research, an instructional design was developed in accordance with Morrison, Ross & Kemp model.
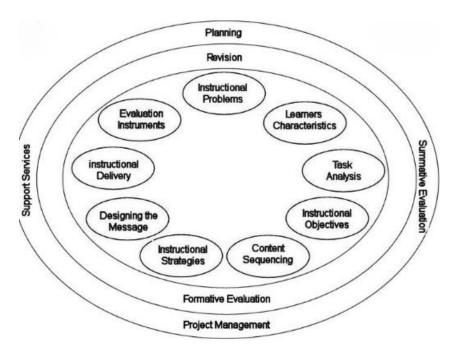
Figure 2. Instructional Design Model of Morrison, Ross & Kemp

Reference: Morrison, Ross & Kemp (2012)

Mathematical Misconception

According to Akkaya (2018), misconception is not a calculation error, lack of information or error. Error is the result of misconception and includes the errors in answers (Baki and Bell, 1997). According to Zembat (2010), misconception cannot be considered without determining the thought behind the source of the error. In this context, misconception can be defined as the perception that creates errors systematically (Smith et al., 1994). If students can explain the accuracy of their mistakes confidentially, it is possible to say that they have misconceptions (Eryılmaz and Sürmeli, 2002).

Zembat (2010) classified misconceptions into four categories as overgeneralization, overspecification, mistranslation, and restricted perception. Zembat (2010) expressed the most common misconception as the category of overgeneralization. Experiences of students play an important role in overgeneralization which can be expressed as spreading a certain rule to other concepts. Misconceptions obtained as result of experiences start establishing logical relationships with previous knowledge (Bilgin and Geban, 2001). Students come to schools with some knowledge they have in their minds. The source of this information can be expressed as the past experiences of students. Çetin (2009) stated that some of the current thinking system of the individuals used in making sense of and expressing life can be inaccurate or incomplete. This can be revealed as the source of misconceptions.

When the literature was reviewed, it was determined that there were studies carried out on eliminating the misconceptions in ordering fractions (Soylu and Soylu, 2005), part-whole relationship in fractions (Altıparmak and Özüdoğru, 2015), representation of fractions on the numerical axis (Pesen, 2008). It has been observed that studies have been conducted to identify and eliminate misconceptions. In addition to sub-topics of

mathematics, it was observed that there were studies carried out on misconceptions in different geometry sub-subjects such as polygons, quadrilaterals, space, point-line-plane, angles on the line, circumference-area-volume calculation (Erbay, 2016; Özkan, 2015; Doyuran, 2014; Doğan, 2013; Başkurt, 2011; Yılmaz, 2011; Dağlı, 2010; Jin and Wong, 2010; Kiriş, 2008; Brinkmann, 2003). It was determined in these studies that different measurement tools were used for identifying and eliminating the misconceptions. Written and oral materials, observation and interview methods, concept cartoons (Kabapınar, 2005), concept maps (Ülgen, 2004), open-ended questions (Eryılmaz and Sürmeli, 2002) were possible to be used for determining the misconceptions. Accordingly, it was also possible to specify that these materials could be used in determining and eliminating the misconceptions in different disciplines and sub-subjects as well as in mathematics.

Number Sense

Although the starting point of number sense has not been known exactly, it was possible to say that it was firstly expressed during the studies of the National Council of Teachers of Mathematics (NCTM, 1989). It was noticed that the researchers had different definitions for the sense of number (Reys and Yang, 1998; Howden, 1989; Kaminski, 2002; Berch, 2005; Howel and Kemp, 2005). According to Dehaene (2001), number sense was based on specially evolved cerebral networks to represent basic arithmetic knowledge. Sowder (1992) made inferences about the number sense and its importance while expressing the skills associated with the ability of prediction. Berch (2005) stated that there were differences between the meaning of number in mathematical cognition literature and the meaning of number in mathematics teaching literature. According to some mathematicians, students' number sense improves as their grade level increases (Sowder, 1992; Yang, 1995). In the book titled Curriculum and Evaluation Standards for School Mathematics, the children with number sense were especially expressed as follows (NCTM, 1989, p.38).

> *Children with the sense of number (1) understand the meanings of numbers very well, (2) develop multiple relationships between numbers, (3) notice the relative magnitude of numbers, (4) understand the effects of operations upon numbers, and (5) develop reference point for measuring the objects around.*

Howden (1989) defined number sense referring the answers of students related to the number 24 as "two tens and four cents," "two dozen of eggs," "6 cents subtracted from 3 tens," "my uncle turned 24 on Saturday," "I will be 24 in 17 years," and "the number 24 is approximately in the middle of the number 20 and 30." Accordingly, the sense of number could be defined as reaching a result using more than one way. Greeno (1991) expressed that number sense included three components as *flexibility in numerical calculations*, *numerical prediction*, and *numerical reasoning and inference*. McIntosh et al. (1992) stated that the number sense included three components as *the concept of number*, *operations with numbers*, and *number and operation applications*. Sowder and Schappelle (1994) made a different distinction and categorized the sense of number under two components as *understanding numbers* and

calculating by rethinking. For example, when making the 26+43 mathematical operation, 20 and 40 were firstly summed and 60 was obtained. Then 6 and 3 were summed together and added to the number 60. The mental process operated here was understanding and re-sensing the numbers.

According to all this information, "perception" is on the basis of mathematical misconceptions. Misperceptions or incomplete perceptions cause misconceptions, and these lead students to make mistakes. Inability to make sense of information depending on perceptions prevents noticing and using different ways of solution. Providing environments and activities to improve the sense of number in individuals as of early childhood prevents mathematical misconceptions (Umay, 2003). Knowing what the number means, interpreting it and reaching conclusions using different ways reduces the errors that appeared due to misconceptions. Carrying out studies on eliminating the misconceptions as one of the basic obstacles for mathematics failure and developing the number sense of individuals should be regarded as the eternal goal of mathematics teaching. Therefore, the number sense-based instructional design developed in this study could be a guide for eliminating the mathematical misconceptions as the responsible of errors in mathematics. Because developed instructional design included cognitive, affective, and psychomotor skills of the students, it was expected to be a guide for teachers in mathematics lessons and have contribution upon curriculum development studies. Furthermore, this research aimed to make mathematics a concrete and fun lesson in which different methods-techniques and materials were used instead of being abstract.

Purpose of the research

In the light of all this information, the purpose of this research was to develop and evaluate a number sense-based curriculum aiming to eliminate student errors due to mathematical misconceptions. In line with this main purpose, answers were sought for the following questions:

1. What were the views of students related to the need for an instructional design in eliminating mathematical misconceptions?
2. What were the student input behaviors to develop an instructional design aiming to eliminate mathematical misconceptions?
3. What was the effect of a number sense-based instructional design upon eliminating misconceptions?

METHOD

Research Model

A mixed method research model as one of the qualitative research approaches was adopted in this study aiming to develop a number sense-based instructional design to eliminate student errors arisen from mathematical misconceptions in secondary school students. According to Cresswell and Clark (2007), data collection, analysis and integration

were performed with qualitative and quantitative methods in mixed method research. If the truth was desired to be understood within a holistic and rich framework, both qualitative and quantitative dimensions should be examined (Yıldırım and Şimşek, 2013). In this study, both qualitative and quantitative methods were employed as it was aimed to collect data on development and efficiency of an instructional design.

Study Group

In preparation stage of the instructional design, 27 seventh grade students studying at a state school in Mersin province were determined as the study group. The research data were collected in 2019-2020 academic year. At the stage of developing and evaluating the instructional design, 24 fifth grade students studying at a state school in Mersin province were determined as the study group. Moreover, in the stage of developing the instructional design, a teachers' board including 4 mathematics teachers and a validity committee including 3 field experts were created. While determining the study group, criterion sampling method as one of the purposeful sampling methods was used. The criteria of the study were determined to have prerequisite knowledge about operations on the number line in terms of students, to have at least 5-year professional seniority in terms of teachers and to carry out studies on the subject in terms of the field experts.

Data Collection Tools

In the process of developing the instructional design, "Needs Analysis Student Interview Form," "Instructional Design Teachers' Board Interview Form," and "Expert Interview Form" were used. Furthermore, "Numerical Axis Operations Skill Form" (NAOSF) was performed to the students for determining student input behaviors and evaluating the efficiency of instructional design. The forms used in the interviews with students and teachers were prepared together with the validity committee.

Numerical Axis Operations Skill Form (NAOSF) used to determine student input behaviors and evaluate the effect of instructional design was developed by the researchers. For the validity study of the form, the opinions of 3 field experts from the Validity Committee were consulted to examine content and language validity. As result of the analyses, the Numerical Axis Operations Skill Form (NAOSF) was used in the pilot implementation with 14 students excluded from the study group. In the Numerical Axis Operations Skill Form (NAOSF) finalized after the pilot implementation, there were number sense-based 40 questions including multiple choice, open-ended, and gap-filling sections. The highest score possible to be obtained from the evaluation form was determined to be 54. One way for determining whether the method used was efficient or not was to analyze its reliability (Jackson, 2008, p. 67). Therefore, the student answers were scored separately by the researchers in line with the rubric prepared previously. The fit relationship between raters was calculated to be .91 using the Pearson Correlation Coefficient. According to Büyüköztürk (2006), the correlation coefficient (r) indicated high correlation level for the

values between .70 and 1.00. A sample question in the *Numerical Axis Operations Skill Form (NAOSF)* is shown in figure 3.

E- **Please sort on the number line of the parameters whose decimal representation is given below.**

Figure 3. A Sample Question in the *Numerical Axis Operations Skill Form*

Data Analysis

Descriptive analysis technique was used for the analysis of the data obtained from the interview forms. Paired groups t-test as one of the parametric tests was used for analyzing the pre-test-post-test score differences obtained from the Numerical Axis Operations Skill Form. In addition, all data were analyzed at macro level after the research was completed. For ensuring coder reliability (Miles and Huberman, 1994), the researchers were allowed to encode independently on the interview texts and the coder reliability was calculated to be .92.

Ethical considerations

A volunteer consent form was presented to parents and teachers on behalf of the students. Throughout the research, the volunteering of the participants was regarded, and the participants were not forced for any implementations. Throughout the implementation, the participants were given right to leave the study.

In this study all rules stated to be followed within the scope of "Higher Education Institutions Scientific Research and Publication Ethics Directive" were followed. None of the actions stated under the title "Actions Against Scientific Research and Publication Ethics", which is the second part of the directive, were carried out. Ethics committee approval was not required as the research data were collected in 2020.

RESULTS

Stages of the Instructional Design

Needs Analysis

For deciding on the subject area of the instructional design, secondary school mathematics course books used in schools were analyzed firstly. In these analyzed textbooks, the subject areas allocated with the most weight and time during the academic

year were priorly determined. Subsequently, the subjects with the highest percentage of questions in the student selection exams for high schools with different names in the last 10 years were determined. In addition, information was collected from the teachers' board about the subjects they allocated the most time while lecturing. As result of these data, student errors arisen from mathematical misconceptions on the subjects related to "Numbers and Operations" learning area from the secondary school 5th grade mathematics curriculum were decided to be analyzed. "Numbers and Operations" was a unit included in each level of the secondary school gradually. Because the acquisitions were most in the fifth-grade curriculum, the sub-learning areas to be used in needs analysis study were chosen from the fifth-grade mathematics curriculum. Data was planned to be collected for deciding on the operations in the sub-learning areas of the 5th grade "Numbers and Operations" learning area to make an instructional design application. The target group was determined to be the students and teachers. At this stage, the seventh-grade secondary school students were primarily interviewed. The reason for choosing the seventh-grade students for the interview was that the "Numbers and Operations" unit in the mathematics curriculum was predominantly included in the fifth and sixth grades. In other words, since the seventh-grade students had detailed information about this unit, it was thought that more reliable answers were possible to be obtained from them to the interview questions. Twenty-seven seventh grade students studying at a public school in Mersin province participated into the interview. The gender distribution of these students was presented in Table 1.

Table 1

Gender Distribution of the Students Interviewed in the Stage of Needs Analysis

Gender	N	%
Female	15	55.6
Male	12	44.4

The interview questions addressed to the students were for determining the sub-learning areas they had the most difficulty in the unit of "Numbers and Operations". The first of these questions was "*What is the most difficult subject you have while learning the numbers and operations unit?*" And the second question was "*What do you think is the reason you have difficulty while learning this subject?*" In order for the answers obtained from the students to be clear, the acquisitions of the unit were written on the board. The frequency distribution of the data obtained from the interview questions was presented in Table 2.

Table 2

The Subjects Students Had the Most Difficulty in Numbers and Operations Unit

Acquisitions	f	%
Indicates and orders unit fractions on the number axis.	9	33.3

Indicates and orders the numbers in decimal notations on the number axis.	8	29.6
Orders the fractions with equal numerators or denominators.	5	18.5
Compares a natural number and a compound fraction.	3	11.1
Finds the elements (multiplier, quotient, or divisor) that were not given in operations understanding the relationship between multiplication and division operations.	1	2.7
Associates a percentile expression with a fraction and decimal notation representing the same size, converts these notations into each other.	1	2.7

According to Table 2, the first of the acquisitions the students participating in the interview had the most difficulty was determined to be "Indicates and orders unit fractions on the number axis" and the second one was "Indicates and orders the numbers in decimal notations on the number axis." This acquisition was complementary to each other. In other words, representation on the number axis was the prerequisite for ordering and fraction knowledge was the prerequisite for decimal notation. As result of the analyses on the data obtained from the interviews, it was concluded that the students had difficulties in indicating the number axis, these difficulties were caused due to misconceptions and lack of sense of number, and the students spent much time in solving the questions about this subject. Subsequent to the needs analysis study, it was decided to develop the instructional design for the subject area of "Numbers and Operations" related to the acquisitions of "Indicates and orders unit fractions on the number axis" and "Indicates and orders the numbers in decimal notations on the number axis." Since the subjects were based on numerical knowledge, the instructional design was planned to be prepared depending on number sense enabling students to use different ways for reaching to solutions making logical inferences. These two acquisitions were combined as *Operations on the Number Axis* to be the instructional design of an operative process. In other words, student errors arisen from mathematical misconceptions were decided to be analyzed on operation skills on the number axis.
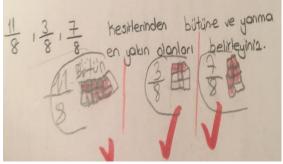
Learner Analysis

In order to collect data for learner analysis, two-hour lessons were lectured with the 5th grade students on the specified subject. In this course, exercises were performed to determine the students' input characteristics, and then the Numerical Axis Operations Skill Form (NAOSF) prepared as a pre-test implementation was employed to the 5th grade students, as the primary target group. This form performed as a pre-test for determining the input behaviors of students was also performed as a post-test for specifying the efficiency

of the instructional design. The information related to the pre-test scores of the students about the operation skills on the number axis was presented in Table 3.
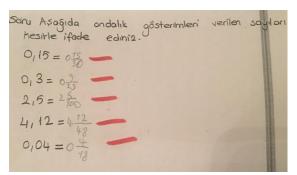
Table 3

Analysis Results Related to the Pre-test Scores of the Students Regarding the Numerical Axis Operations Skill Form

Number of Participants	Mean	Standard Deviation	Mod	Median	Min Score	Max Score	Max Score Possible to	Skewness	Kurtosis
24	13.66	7.12	10	12.5	3	30	54	1.11	.87

As presented in Table 3, the arithmetic mean of the pre-test scores of the participants related to Numerical Axis Operations Skill Form before the implementation process was 13,66. Because the highest score possible to be taken from the Numerical Axis Operations Skill Form was 54, the success levels of the participants in terms of input behaviors were possible to be mentioned as low. Some of the student answers given to the Numerical Axis Operations Skill Form were presented in Figure 3 and Figure 4.

Figure 4. Student Answers Related to Input Behaviors Form-1

Figure 5. Student Answers Related to Input Behaviors Form-2

According to Figure 3, the students had misconceptions confusing on proximity to the whole and half of the multiplicities shaped as fractions. Errors were determined in student answers due to misconceptions. According to Figure 4, the misconception indicating the inability of making transfers between decimal representation and fraction representation was remarkable. A group assessment was made with four mathematics teachers with 7 to 10-year professional seniority on student answers. The common view of the group teachers was as follows:

> *"Answers of the students were meaningful. Particularly, while lecturing the indication of fractions and decimal notations on the number axis and operations in the lesson, more questions were asked by the students and it was observed that more errors were made in these subjects in assessment process. It was observed that the students had misconceptions arisen from lack of conceptual knowledge about the subject. They thought that the numbers in the*

numerator and denominator of the fraction came from a different number system." (Branch Teachers Board)

The information related to the target group administered with instructional design was as follows:

First Target Group (The implementation was performed for these students.)

- Secondary school 5th grade students

Secondary Target Group (The implementation will be voluntary for these students. The students who feel inadequate on this subject could participate)

- Secondary school 6th grade students

General learner characteristics

- Age: 10-11
- Gender: 46% (f=11) female, 54% (f=13) male (first target group)
- Level of literacy: All students had literacy skills (first target group)
- Socioeconomic levels of the students were medium and over (first target group)
- The mostly loved courses were Mathematics, Physical Education, Turkish, respectively (first target group)

Input characteristics

- It was observed that they had difficulty in establishing the relationship between the fraction number corresponding to a certain multiplicity and this multiplicity.
- Errors on using numerator and denominator interchangeably was observed due to not comprehending the part-whole relationship fully.
- Misconceptions were observed while classifying fractions as proximity to the whole and proximity to the half.
- The students were observed to follow a single way while establishing a relationship of size or equality between decimal representations.

Common errors of the target group:

- The students had difficulty in creating the denominator while writing the numbers with decimal notation as fractions.
- The students fell into misconceptions not being able to determine what decimals meant as numbers.

Academic knowledge

- Previous year grades of the learners for the mathematics course were medium and high.
- General grade averages of the learners related to the previous year were low, medium, and high.
- Previous year grades of the learners for other relevant courses were medium and high.

It was determined that the errors were the result of misconceptions arisen from the lack of sense of number. An example of student errors appeared depending on the observed misconceptions was presented in Figure 5.
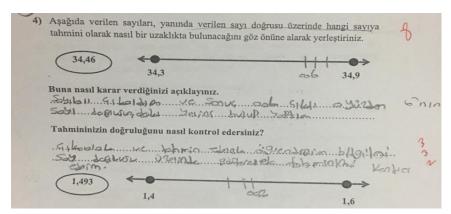
Figure 6. The error made as result of misconception based on sense of number

According to Figure 5, the student was asked to place the given decimal notation in approximately the correct place in the interval shown on the number axis. It was possible to say that there was a misconception related to the decimal part that came after the full part of the decimal notation. The basis for this misconception was the inability of understanding and perceiving the number. Whereas the decimal representation of 34.46 should be closer to 34.3, the lack of number perception in the student prevented him/her from obtaining conclusions using different ways and caused him/her to make mistake driving to misconception.

Context Analysis

The information related to the contextual factors to be considered for the development and implementation stages were as follows:

Guiding context

- Why did you want to learn this course?

The learners stated that they wanted to learn this course because mathematics was used in every profession and situation and it would be useful in their daily life, shopping.

- How could you benefit from the information you learned in this lesson?

The learners stated that they could benefit from this information in their other courses, daily life, and mathematics lessons.

- What kind of acquisitions you expected at the end of this course?

The learners stated that they could get high grades from the mathematics course, they could get certificates of appreciation or excellence, and they could choose the profession they wanted answering the math questions correctly in the exams.

Transfer Context

- How did you apply the knowledge and skills you learned in similar or different situations?

The learners stated that they had the opportunity of applying the knowledge they acquired in mathematics in many courses, especially in Science. They stated that they could understand some expressions they could not make sense on TV.

- Where and how could you use this information?

The learners stated that they could use this information in their daily lives, family conversations, at school, and answer questions mathematically correctly.

- What conditions did you think were necessary for you to use these information and skills?

The learners stated that there should be conditions and environments such as a good family environment, school environment, working environment, and competitions.

Learning-Teaching Context

The information related to learning-teaching environments was presented in Table 4.

Table 4

The information related to learning-teaching environment

Factor	Points to be considered
Lighting	Since the classroom was on the top floor, there was no problem with lighting. Sunlight and lighting were benefited.
Noise	Since the classroom was on the top floor, the noise outside the school was not heard from the classroom.
Temperature	Since there was no air conditioning in the classroom in summer, the temperature increased, and the ideal temperature was reached in winter when the radiators burnt out.
Order of seating	The order of seating in the classroom was standard classroom seating with three groups. Two students sat in each desk. Their desks were separate, but the tables were common.
Material equipment	There were a board, smart board, cupboard, and recycling bin in the classroom, and the materials were brought to the lesson from the material room when needed. However, new generation materials aren't used for maths course.
Transportation	The school had a central location. It was on the bus and subway route.

Content Analysis

The content analysis related to the subject was presented below.

Prerequisite knowledge:

- It was necessary to have knowledge on whole number, natural number, cardinal number, and concept of number.
- It was necessary to have a sense of number and use it in transactions.

Factual knowledge:

- Z: Set of whole numbers
- /: Fraction bar

Concepts:

Fraction, rational number, unit fraction, simple fraction, compound fraction, mixed fraction, number axis, decimal notation

The concept analysis created by the field experts for the concepts was presented in Table 5.

Table 5

Concept Analysis

Name of the concept	Fraction
Definition	Fractions are numbers representing one or more of the equal parts into a unit is divided. Because a whole cannot be negative, fractions cannot take negative values. * The denominator indicates how many equal parts a whole is divided into, and the numerator indicates how many of these parts are taken or scanned. * The division of a number by zero is undefined. Since we cannot divide a whole into zero parts, there cannot be zero in the denominator.
Distinctive features of the concept	a and b must be integers each and b must be different from zero (a/b) Fractions cannot take negative values.
Nondistinctive features of the concept	Having numerator, denominator and fraction bar, denominator being a non-zero number
Examples of the concept	$\frac{1}{2}, \frac{6}{7}, \frac{12}{4}$
Non-examples of the concept	$\frac{3}{0}, \frac{5}{0}, \frac{8}{0} - \frac{4}{8}, -\frac{5}{11}$
Name of the concept	Rational number
Definition	There are two different ways of indicating rational numbers as fractions and decimals. However, the point to be noted here is that the fraction representation of rational numbers is a special notation, that is, it includes the part that cannot be represented negatively. Fraction notation is used as the numerical equivalent of the representations to show only certain parts of a whole.
Distinctive features of the concept	It includes the equivalence class. It can take negative and positive values. Special notations are possible with fractions and decimals on it.

Nondistinctive features of the concept	Having numerator, denominator and fraction bar, denominator being a non-zero number
Examples of the concept	$\frac{1}{2}, \frac{2}{4}, \frac{6}{12}, \frac{-2}{3}, -\frac{-4}{6}, -\frac{10}{15}$
Non-examples of the concept	$\frac{3}{0}, \frac{5}{0}, \frac{8}{0}$
Name of the concept	Unit fraction
Definition	Simple fractions with a numerator of 1 representing only one of the equal parts of a whole.
Distinctive features of the concept	Unit fractions belong to the simple fraction type. They refer to only one of the equal parts of a whole.
Nondistinctive features of the concept	Unit fractions are expressed as fractions notation. They have a numerator and denominator. They can be represented on the number axis.
Non-examples of the concept	$\frac{3}{5}, \frac{6}{7}, \frac{9}{5}, 2\frac{3}{5}$
Name of the concept	Simple fraction
Definition	The fraction with a numerator that is lower than denominator and can take values between 0 and 1.
Distinctive features of the concept	Their numerator should be less than their denominator. They are located between 0 and 1 on the number axis.
Nondistinctive features of the concept	They can be represented on the number axis. They are expressed as fraction notation.
Examples of the concept	$\frac{1}{2}, \frac{6}{7}, \frac{2}{3}$
Non-examples of the concept	$\frac{3}{2}, 2\frac{5}{7}, \frac{7}{3}, \frac{15}{15}$

Name of the concept	Compound fraction
Definition	The fractions with numerator which is equal or greater than its denominator.
Distinctive features of the concept	The numerator must be greater than or equal to the denominator. Because the numerator is equal to the denominator, the fraction is equal to integer 1, and when the numerator is higher than the denominator, it is higher than 1.
Nondistinctive features of the concept	They can be represented on the number axis. They are expressed as fraction notation.
Examples of the concept	$\frac{3}{2}, \frac{10}{7}, \frac{9}{5}$
Non-examples of the concept	$\frac{1}{3}, \frac{3}{5}, \frac{2}{7}$
Name of the concept	Mixed fraction
Definition	Fractions that add one or more whole to simple fractions and are a special representation of compound fractions.
Distinctive features of the concept	It is the special form of compound fractions. Mixed fractions can be converted into compound fractions, and compound fractions can be converted into mixed fractions.
Nondistinctive features of the concept	They can be represented on the number axis. They are expressed as fraction notation.
Examples of the concept	$2\frac{6}{7}, 1\frac{9}{5}$
Non-examples of the concept	$\frac{1}{3}, \frac{3}{5}, \frac{2}{7}$

Name of the concept	Number axis
Definition	It is the most basic coordinate system where each point corresponds to a related real number and the distance between the points is equal.
Distinctive features of the concept	The points including whole numbers, these are shown with equal distance between them. It is double-sided as negative and positive. It can be drawn horizontally as well as vertically.
Nondistinctive features of the concept	It is a special coordinate system. Each number has a place on the number axis.
Examples of the concept	Lifts, thermometers
Non-examples of the concept	Ruler, scissors, clock

Name of the concept	Decimal notation
Definition	It is the representation made using comma/dot between the whole and less than one complete parts of numbers on which four operations can be performed.
Distinctive features of the concept	There is no such thing as a decimal number, but the numbers have decimal notation. When a whole is divided into 10, 100, or 1000 equal parts, then the units of the resulting fraction are expressed in decimal notation. Decimal notations include the whole part and the fraction part. Size relationships can be determined comparing the numbers with decimal notation.
Nondistinctive features of the concept	Numbers are used to write decimal notations. Decimal notations can be expressed as rational numbers. Because it is a special representation of rational numbers.
Examples of the concept	3.5 – 45.24 – 0.7 – 35,313 – 201,495 – 5,0
Non-examples of the concept	$\sqrt{7}$, 5i, $\sqrt[3]{6}$

Operational knowledge:

While the fractions were presented on the number axis,

- It was decided which two integers the fraction was between and which integer it was closer to with the sense of number.
- This specified range was divided into equal parts as many as the number in the denominator.
- Starting to count from the left side, the numerator was counted and marked until it reached that number.

While the decimal notations were expressed on the number axis,

- It was decided which two integers the decimal notation was between and which integer it was closer to with the sense of number.
- On the specified range, the range of the numbers in the digits after the decimal notation was determined.
- This specified range was divided into 10 equal parts.
- Starting to count from the left, it was counted until the next digit after the comma and it was marked.

The principles:

The fractions got lower as they approached to 0 on the number axis and got greater as they approached to 1. In decimal notations, the fraction with the greater whole part is greater; if the whole parts were equal, the digits after the comma were regarded at respectively and decided according to this rule.

Target Analysis

Unit title/Subject: Fractions/Algebra

Subject: Operations on the number axis

Grade: 5

Number of courses: 10

In the instructional design to be prepared, the subject of "Operations on the Number Axis" was planned to be completed in 10 course hours. In the needs analysis study and interviews with subject area experts, it was determined that the acquisitions related to the subject should be clear and understandable, the subjects as the continuation of each other should not be fragmented, and the achievements above the student level should not be included. For this reason, the subject of "Operations on the Number Axis" was determined as the planned subject of the instructional design, and the operations of presenting the fractions and decimal notations on the number axis as a continuation of each other was included. In accordance with this purpose, the following acquisitions was used in the instructional design to be prepared for "Operations on the Number Line:"

- Indicated between which two integers a given fraction was on the number axis. (2*)
- Showed and ordered unit fractions on the number axis. (2*)
- Showed proper, compound and integer fractions on the number axis and interpreted the highness and lowness relations between them. (2*)
- Expressed the numbers given as decimal notation as fractions and fractions as decimal notation. (2*)
- Showed and ordered the numbers given as decimal notation on the number axis. (2*)

*Number of courses

The cognitive process and knowledge dimension of the acquisitions in the current program were presented in Table 6 and Table 7.

Table 6

Cognitive Process Dimension Related to Acquisitions

Acquisitions	Cognitive Process Dimension					
	Reminding	Understanding	Implementing	Analyzing	Evaluating	Creating
Indicates between which two integers a given fraction is on the number axis.			X			
Shows and orders unit fractions on the number axis			X			
Shows proper, compound and integer fractions on the number axis and interprets the highness and lowness relations between them.			X			
Expresses the numbers given as decimal notation as fractions and fractions as decimal notation.		X				
Shows and orders the numbers given as decimal notation on the number axis.			X			

Table 7

The Dimension of Knowledge Related to Acquisitions

Acquisitions	The Dimension of Knowledge			
	Factual Knowledge	**Conceptual Knowledge**	**Operational Knowledge**	**Metacognitive Knowledge**
Indicates between which two integers a given fraction is on the number axis.		X		
Shows and orders unit fractions on the number axis			X	
Shows proper, compound and integer fractions on the number axis and interprets the highness and lowness relations between them.			X	
Expresses the numbers given as decimal notation as fractions and fractions as decimal notation.		X		
Shows and orders the numbers given as decimal notation on the number axis.			X	

Content Editing Strategy

Message design was used as the content editing strategy in the developed instructional design. As a message design, a written text was developed for the subject of "Operations on the Number Axis."

Implementation and Assessment

Ten course hours were lectured in accordance with the instructional design prepared with the 5th grade students. In order to determine the efficiency of the instructional design at the end of the application, the *Numerical Axis Operations Skill Form (NAOSF)* was performed as a post-test to the 5th grade students as the primary target group. Subsequently, whether there was a significant difference between the pre-test and post-test mean scores of the participants was analyzed. The analysis results were presented in Table 8.

Table 8

Pre-test and Post-test Paired Groups T-Test Results Related to Students' Operational Skills in Number Axis

Measurement	Number of Participants (N)	Arithmetic Mean (X)	Standard Deviation (S)	Degree of Freedom (Sd)	t Value	Level of Significance (p)
Pre-test	24	13.66	7.11			
Post-test	24	30	9.18	23	-17.277	.000*

*p<.001

As could be seen in Table 8, a statistically significant difference was determined between the pre-test and post-test scores of the participants (p<.001). This difference was in favor of the post-test scores. According to this result, it could be said that students' errors in number axis operation skills decreased as result of the instructional design based on number sense.

DISCUSSION

A needs analysis was primarily performed in this study with the main purpose of developing a number sense-based instructional design to eliminate student errors due to mathematical misconceptions. Subsequent to the studies carried out within the scope of the needs analysis, it was decided that the sub-topic to be developed in the instructional design would be the order and representation of fractions and the numbers given in decimal representation on the number axis.

Within the scope of needs analysis, student views were asked for an instructional design need to eliminate student errors arisen from mathematical misconceptions. According to the interviews with the seventh-grade students, the sub-learning areas the students had the most difficulty were "*indicates and orders unit fractions on the number axis*," and "*indicates and orders the numbers given in decimal representations on the number axis*," respectively. According to the analysis results of the answers given by the students to the reasons for having difficulty while learning these subjects, it was concluded that there were misconceptions in the concept of number axis which was the prerequisite for ordering and the operational processes related to this concept. It was seen that the students had difficulty in showing fractions and decimal notations. It was determined that there were concept misconceptions about which whole number the fractions and decimal notations would be closer to. In the interviews made with the teachers' board on student responses, it was determined that these misconceptions were caused due to the inability of using the number sense in operations related to fraction and decimal notation. When the literature was reviewed, it was noticed that different methods were used to determine the misconceptions within the scope of techniques (Ang & Shahrill, 2014; Paul & Hlanganipai, 2014; Sarwadi & Shahrill, 2014) and there were researches for eliminating the misconceptions (Durkin & Johnson, 2015; Ojose, 2015; Jankvist & Niss, 2018). Since the common feature of this study and the studies reviewed in the literature was the presence of misconceptions in students

and determining the errors related to this, this result obtained from the study was consistent with the results of the other researches. On the other hand, it was determined in some studies that the number of students who were able to use the number sense as the cause of mathematical misconceptions was low (Yang & Li, 2008; Singh, 2009; Mohamed & Johnny, 2010). In the study carried out by Filiz and Moralı (2020), it was concluded that the students used the rules they learned previously and did not prefer using an alternative way. These results proved that number sense was very important for mathematics education and benefiting from the instructional approaches based upon number sense could be efficient upon decreasing the errors arisen from misconceptions. It could be efficient on using instructional approaches based on number sense to reduce errors due to misconceptions.

As result of the learner analysis performed for determining the input behaviors of the students, it was determined that the skills of the students in the mathematics sub-topic of "Operations on the number axis" which was spirally lectured at different grades were found to be low. Misconceptions arisen from lack of perception led students to make mistakes. For that reason, misconceptions should be identified and eliminated in order to determine learning conditions of the students accurately (Osborne and Gilbert, 1980; Resnick et al., 1989). According to Stefanich and Rokusek (1992), the teaching conducted for this purpose not only eliminated misconceptions but also contributed upon facilitating learning and ensuring the permanence of what was learned. In this research, it was determined that the students had imperceptions related to the position of numbers on numerical axis. This lack of perception caused them to have difficult in the topics in relation to closeness of fractions to the half or the whole and the closeness of decimal notations to the whole numbers.

Subsequent to the needs and learner analyses, context, content, and target analysis studies were included. The instructional design prepared determining the content editing strategies was performed to the students who were the primary target group, and the measurement tool developed by the researchers was performed as the post-test for evaluating the instructional design. According to the analysis results for the pre-test and post-test score comparison of the students, there was a statistically significant difference in favor of the post-test. Instructional designs were remarkable in terms of revealing the paths to be followed in reaching the goal and guiding the process. It was concluded that the instructional design used in the study of Apino and Retnawati (2017) improved high-level thinking skills of the students in learning mathematics. Bennison et al. (2020) concluded that using instructional design could be benefited to support professional learning in arithmetic in mathematics. Similar results were also obtained in the research of Avcu and Er (2020), and it was concluded that the instructional design developed according to the instructional design model of Morrison, Ross & Kemp was efficient upon the digital thinking and creative thinking skills of the students. This situation may result from the fact that the development of instructional design requires progressing by conducting multifaceted analyses. The fact that conducting the detailed analysis of all of the components which were included in the

process by determining the needs positively contributed to the fact that the subject for which the instructional design would be developed were dealt with in depth and therefore the permanence of learning increased.

LIMITATIONS AND RECOMONDATIONS

Depending upon the results obtained from the study, it was possible to suggest that further researches should be conducted for determining the misconceptions of the students related to different mathematics sub-topics. Furthermore, in-service training could be given to teachers who were the practitioners of the curriculum in the classroom for improving their sense of number and eliminating mathematical misconceptions.

CONCLUSION

When all these results were evaluated together, it would be correct to say that number sense was an important prerequisite for the interpretation and use of mathematical knowledge, and it would be efficient on eliminating errors arisen from the mathematical misconceptions when it was integrated into an effective instructional design. Teaching by paying attention to not only the concept of number but also where and how it is used will contribute the fact that the sense of number will develop and the concept misconceptions will be prevented.

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